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Design of vertical drain installations based on darcian and non-darcian flow: A comparison of real behaviour and theory

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SYNOPSIS. Land reclamation works and the need of improving infrastructure in areas with bad soil conditions have created an increasing interest in soil improvement. Among the methods applied, preloading in combination with vertical drain installations to speed up the rate of consolidation has proved to be very cost-effective. The interest in the possibilities of making accurate predictions of the consolidation process achieved by the method has manifested itself in a great number of papers as well as a number of Doctoral theses. In this paper, a short historical review of the theoretical development of the vertical drain theories is presented. The main interest is focused on the fundamental question of validity or non-validity of Darcy's flow law in the vertical drain analysis. A generalised consolidation theory of vertical drains, valid for both exponential (non-Darcian) and linear (Darcian) flow laws, and including the effect of both smear and well resistance, is presented. Vertical drain parameters to be applied, such as the equivalent diameter of prefabricated band (wick) drains, diameter and characteristics of the zone of smear, filter requirements, well resistance, etc. are discussed. Results of full-scale tests and theoretical analyses, based on Darcian as well as non-Darcian flow, are exemplified. The method of analysis of vertical drain projects to be used under more complicated loading conditions is demonstrated. The comparisons undertaken between real and theoretical behaviour in the consolidation process show a better agreement for non-Darcian than for Darcian flow. Only in cases of excessive hydraulic gradients arisen from very heavy loading can we find equally good (or possibly better) agreement between observations and theories based on Darcian flow on one hand and theories based on non-Darcian flow on the other.

1. INTRODUCTION

A wealth of information on the historical background of vertical drain design can be found in a book written by Magnan (1983). In this paper a brief review is given in retrospect of various analytical methods for determining the influence on the consolidation process of vertical drain installations. Design methods based on both Darcian and non-Darcian flow laws are presented. Requisite properties of prefabricated band drains are commented on. Monitoring systems for a follow-up of results achieved are discussed. Results of full-scale tests on vertical drain behaviour are compared with theoretical consolidation rates.

2. VARIOUS TYPES OF VERTICAL DRAINS

2.1 Circular-Cylindrical Drains

Sand drains which are the most common type of circular-cylindrical drains were first proposed, in 1925, and patented, in 1926, by Daniel D. Moran. He also suggested the first practical application of sand drains as a means of stabilisation of mud soil beneath a roadway approach to the San Francisco Oakland Bay Bridge (Johnson, 1970). This led to some successful laboratory and field experiments followed by the installation of the first drain system in 1934. Porter (1936) described these trials and contributed to the further use and development of the system.

The sand drains originally installed had generally a relatively large diameter, 0.3–0.5 m. Later on small-diameter sand drains have come into use, for example 'sandwicks', 0.05 m in diameter, and 'fabri pack drains'—also called 'sand pack drains'— 0.12 m in diameter. The sand in these drains is packed into a synthetic fiber net-type tube which prevents the drains from necking. Sand drains with a diameter of 0.18 m were utilised in the oldest and best documented test field existing, the one situated at Skå Edeby, Sweden, established in 1957 (Hansbo, 1960). This test field is still under continuous observation.

Another type of circular-cylindrical drains was developed by, among others, Technique Louis Ménard, the so-called 'soil drain'. This consists of an open prefabricated tubular plastic core provided with perforations to admit inflow of pore water.

A range of techniques has been utilised for installation of sand drains. These include so-called non-displacement methods, such as shell and auger drilling, powered auger drilling, water-jetting, flight augering and wash-boring and displacement methods, typically by the use of a driven mandrel.

2.2 Band (Wick) Drains

The first type of band-shaped drains, also named wick drains, introduced on the market was invented in Sweden by Walter Kjellman and his co-workers at the Swedish Geotechnical Institute. These drains, named Cardboard Wicks (Kjellman, 1948), were made of two cardboard sheets glued together with an external cross-section of 100 mm by 3 mm and including ten longitudinal internal channels, 3 mm in width and 1 mm in thickness. The efficiency of Cardboard Wicks was first investigated in 1945 in a full-scale test near Upplands Väsby, north of Stockholm, as preliminary measure to the construction of the new Stockholm airport. The results of this investigation was reported by Chang (1981).

The Cardboard Wick has served as a prototype for all the various band-shaped drains now existing on the market. The first of these new types of band drains on the market was named Geodrain, developed at the Swedish Geotechnical Institute. It consists of a core of plastic material surrounded by a filter sleeve with an external cross-section of 95 mm by 4 mm. Both sides of the core are provided with 27 longitudinal grooves whose widths and depths vary with different makes. The filter sleeve was originally made of a special make of paper but was later changed into synthetic material. After some successful applications of Geodrains, a great number of band drains, having more or less similar characteristics but different drainage efficiencies, have been developed (see e.g. Hansbo, 1986, 1993, 1994). Although most band drains have a central core enclosed in a filter sleeve, drain types without filter sleeve also exist on the market. These generally consist of porous material which allows water inlet into the drains. A somewhat different type of band drain is the Fibredrain developed in Singapore (Lee *et al.*, 1995). It consists of one layer of thin, closely knit jute burlap laid inside another layer of thick, but coarsely knit burlap. Four coir strands, 3–6 mm in diameter, pass longitudinally through the inner core formed by the two layers of burlap.

It is interesting to note that Barron (1948), with reference to a contribution by Kjellman (1948), expresses his opinion that "should wick material and installation machines become available in the United States, sand wells may be outmoded". Nowadays very efficient installation machines have come into use and a large number of various band (wick) drains exist on the market. Barron's prophecy has certainly become true.

3. ANALYTICAL APPROACH

3.1 Assumptions based on Darcy's flow law

Regarding the historical development of vertical drain analysis, special interest must be devoted to the contributions given by Barron which form a starting point in the understanding of the result to be expected by vertical drain installations. During the winter 1941–1942 the Providence District incorporated drain wells in plans for reconstruction of a portion of Riverfront Dike, Hartford, Connecticut. This entailed a necessity of having a more exact analysis of the influence of vertical drains on the consolidation process. The analysis first published by Barron (1944) was based on existing solutions for one-dimensional vertical consolidation (Terzaghi, 1925) and radial flow of heat. Barron's analysis was based on the following assumptions (Fig. 1):

- Darcy's flow law is valid,
- the soil is water saturated,
- displacements due to consolidation take place in the vertical direction only,
- excess pore water pressure at the drain well surface is zero,
- the cylindrical boundary of the soil mass is impervious, i.e. $\partial u/\partial \rho = 0$ at $\rho = R$.
- excess pore water pressure at the upper boundary of the soil mass ($z = 0$) is zero.
- no vertical flow at the central horizontal boundary of the soil mass, i.e. $\partial u/\partial z = 0$ at $z = l$.

The differential equation governing the consolidation process is then given by the expression:

$$\frac{k_h}{\gamma_w}\left(\frac{1}{\rho}\frac{\partial u}{\partial \rho}+\frac{\partial^2 u}{\partial \rho^2}\right)+\frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2}=\frac{a_v}{1+e}\frac{\partial u}{\partial t} \tag{1}$$

where k_h and k_v are the permeabilities in the horizontal and vertical directions,
γ_w = the unit weight of water,
ρ and z are the cylindrical coordinates,
u = excess pore water pressure,
$a_v = -\Delta e/\Delta\sigma'$ = coefficient of theoretical compressibility
e = void ratio
$a_v/(1+e) = m_v = 1/M$ = coefficient of volume compressibility (M = oedometer modulus),
t = consolidation time.

Barron proposes that the total degree of consolidation, including the effect of combined radial and vertical outflow of water, be solved according to Carillo (1942) by the expression:

$$u_{\rho z}=\frac{u_\rho u_z}{u_0} \tag{2}$$

where $u_{\rho z}$ = remaining total excess pore water pressure after time t,
u_ρ = remaining excess pore water pressure after time t due to radial drainage,
u_z = remaining excess pore water pressure after time t due to vertical drainage,
u_0 = excess pore water pressure at time $t = 0$.

Expressed in degree of consolidation $U = 1 - u/u_0$ this yields:

$$U_{\rho z}=U_\rho+U_z-U_\rho U_z \tag{3}$$

where U_ρ = degree of consolidation due to radial outflow of pore water to the drains,
U_z = degree of consolidation due to vertical outflow of pore water outside the drains.

As an alternative to $U = 1 - u/u_0$ we can also use the definition $U = s/s_p$, where s = settlement at time t and s_p = total primary settlement. Then the settlement s_{hd} at time t, achieved by the effect of radial drainage only, can be written

$$s_{hd}=\frac{s-s_{vd}}{1-s_{vd}/s_p} \tag{4}$$

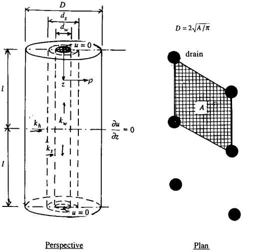

Fig. 1. Terms used in the analysis of vertical drains: D = diameter of soil cylinder dewatered by a drain, d_s = diameter of the zone of smear, d_w = drain diameter, z = depth coordinate, l = length of drain when closed at bottom ($2l$ = length of drain when open at bottom), q_w = specific discharge capacity of the drain (vertical hydraulic gradient inside the drain $i = 1$).

where s_{vd} = settlement caused by one-dimensional vertical consolidation.

Barron assumed two different cases to take place: the case of *free strains* and the case of *equal strains.*

In the *free strain hypothesis* Barron assumes that the load is uniform over the circular zone of influence for each drain well and that differential settlements occurring over the zone during the consolidation process have no effect on redistribution of stresses by arching of the fill. He further assumes that shearing strains caused by differential settlements have no influence on the consolidation process.

In his original free strain analysis, Barron (1944) assumed that the installation of the drains did not affect the properties of the soil and that the permeability of the drain well was high enough for well resistance to be neglected. He later on included disturbance effects due to installation, a *zone of smear* (Barron, (1948) with reduced permeability k_s. Assuming that drainage takes place only in the radial direction and that the initial excess pore pressure u_0 is constant throughout the soil, the excess pore pressure at any point ρ, thus derived by Barron, is found from the relation:

$$u_\rho=u_0\sum_{a_1,a_2,\cdots}^{\infty}\frac{-\frac{2d_w}{ad_s}U_1(ad_s/d_w)U_0(2a\rho/d_w)}{\frac{4d_w^2}{\pi^2a^2d_s^2}-U_0^2(ad_s/d_w)-U_1^2(ad_s/d_w)}\exp\left(-\frac{4a^2c_ht}{d_w^2}\right) \tag{5}$$

where $U_0(ad_s/d_w)=J_0(ad_s/d_w)Y_1(aD/d_w)-J_1(aD/d_w)Y_0(ad_s/d_w)$,
$U_1(ad_s/d_w)=J_1(ad_s/d_w)Y_1(aD/d_w)-J_1(aD/d_w)Y_1(ad_s/d_w)$,
$U_0(2a\rho/d_w)=J_0(2a\rho/d_w)Y_1(aD/d_w)-J_1(aD/d_w)Y_1(2a\rho/d_w)$
$J_0(\ )$ and $J_1(\ )$ are Bessel functions of first kind of zero and first order, respectively,
$Y_0(\ )$ and $Y_1(\ )$ are Bessel functions of second kind of zero and first order, respectively,
$a_1, a_2, \cdots$ are roots of the equation $\frac{k_sd_wU_0(ad_s/d_w)}{k_had_s\ln(d_s/d_w)}+U_1(ad_s/d_w)=0$
k_h = permeability of the soil in the horizontal direction,
k_s = permeability in the zone of smear,
$c_h = k_h/m_v\gamma_w = k_hM/\gamma_w$,
$1/m_v = M$ = oedometer modulus (inverted coefficient of volume compressibility),
γ_w = unit weight of water,
$d_w = 2r_w$ = diameter of the drain well,
$D = 2R$ = diameter of the dewatered cylinder,
d_s = diameter of the zone of smear,
t = time of consolidation.

The average excess pore water pressure between r_w and R becomes:

$$\bar{u}=u_0\sum_{a_1,a_2,\cdots}^{\infty}\frac{d_w^2U_1^2(ad_s/d_w)\exp\left(-4a^2c_ht/d_w^2\right)}{a^2\left(D^2-d_s^2\right)\left[\frac{4d_w^2}{\pi^2a^2d_s^2}-U_0^2(ad_s/d_w)-U_1^2(ad_s/d_w)\right]} \tag{6}$$

The radial consolidation rates obtained by equations (5) and (6) are exemplified in Fig. 2. As can be seen in Fig. 2 — and this holds for any value of D/d_w — the average excess pore water pressure during the course of consolidation is very nearly equal to the excess pore water pressure at radius $\rho = (r_w + R)/2$.

In the *equal strain hypothesis* Barron (1948) presumes arching to redistribute the load so that the vertical strains at a certain depth z become equal irrespective of the radial distance ρ, and, consequently, no differential settlement will take place. This may seem a rather serious condition but is supported by field observations in areas provided with vertical drains. In the equal strain hypothesis he also includes the effect of *well resistance* on the consolidation process. Thus, in reality the drains may have a limited capacity of transporting the pore water entering into the drains during the consolidation process. Assuming complete drainage at $z = 0$ and $z = 2l$ the average degree of consolidation obtained by radial (horizontal) drainage $\overline{U}_{hz} = 1 - \overline{u}_{hz}/\overline{u}_0$ at depth z is given by the correlation (Fig. 1):

$$\overline{U}_{hz} = 1 - \exp\left\{-\frac{8c_h t}{vD^2}\left[\frac{\exp[\beta(z-2l)] + \exp(-\beta z)}{1+\exp(-2\beta l)}\right]\right\} \tag{7}$$

where $v = \frac{D^2}{D^2 - d_s^2}\ln\left(\frac{D}{d_s}\right) - \frac{3}{4} + \frac{d_s^2}{4D^2} + \frac{k_h}{k_s}\left(\frac{D^2 - d_s^2}{D^2}\right)\ln\left(\frac{d_s}{d_w}\right)$

k_h, k_s, c_h ····as above,

$$\beta = \sqrt{\frac{8k_h\left(1 - d_s^2/D^2\right)}{d_w^2 k_w v}} = \sqrt{\frac{2\pi k_h\left(1 - d_s^2/D^2\right)}{v q_w}}$$

$q_w = k_w \pi d_w^2/4$ = specific discharge capacity of the drain

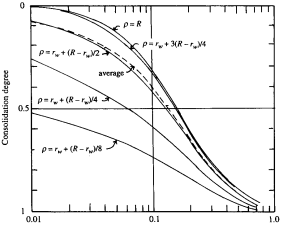

Fig. 2. Radial consolidation rates at different concentric surfaces according to Barron's free strain hypothesis. Ideal drain wells (no well resistance). No effect of smear. $D/d_w = 10$. After Barron (1944).

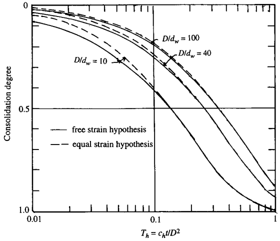

Fig. 3. Comparison of average consolidation rates by radial drainage only for various values of D/d_w under conditons of equal vertical strains at any given time and no arching of overburden (free strain hypothesis). Ideal drain wells. No effect of smear. After Barron (1948).

The average degree of consolidation obtained for ideal drains according to Barron's free strain analysis, equation (6), is very nearly equal to that obtained according to Barron's equal strain analysis, equation (7), as shown in Fig. 3. Therefore, the equal strain hypothesis has become the basis for routine design of vertical drain systems.

The consolidation equation presented by Barron is based on the assumption that the value of c_h does not change during the consolidation process. An attempt to take into account a successive decrease of of c_h with time of consolidation, expressed through a decrease in the coefficient of permeability, was made by Schiffman (1958). Schiffman's concept is based on a linear correlation between permeability coefficient and excess pore water pressure. For constant load, varying permeability (oedometer modulus assumed to remain constant during the consolidation process) and radial drainage only, the solution obtained for ideal drain wells (no well resistance; no smear) becomes:

$$\overline{U}_h = 1 - \frac{k_f}{k_f + k_0\left[\exp(8T_f/\upsilon) - 1\right]} \tag{8}$$

where k_f = final value of k_h
k_0 = initial value of k_h
$T_f = c_f t/D^2$
$c_f = k_f M/\gamma_w$

$$\upsilon = \frac{D^2}{D^2 - d_w^2}\ln\left(\frac{D}{d_w}\right) - \frac{3}{4} + \frac{d_w^2}{4D^2}$$

The parameter υ is equal to the parameter v in equation (7) when $d_s = d_w$.

The effect of well resistance was also taken into account by Yoshikuni and Nakanado (1974). Their solution, which includes both vertical and horizontal pore water flow (upper and lower boundary surfaces assumed to be drained) but does not include the effect of smear, ends up in a rather complex expression. The results obtained in the case of radial drainage and equal vertical displacement of the soil surface (which is not exactly synonymous with the equal strain case), are presented in the form of tables (see Yoshikuni, 1992) for various values of D/d_w and well resistance, the latter expressed by the parameter:

$$L = \frac{32 k_h l^2}{\pi^2 k_w d_w^2} = \frac{8 k_h l^2}{\pi q_w}$$

Youshikuni's solution, which was presented in his Doctoral thesis in 1979 (in Japanese), was extended by Onoue (1988) to include the influence exerted on the consolidation process by multilayered, unisotropic soils.

Another approach to the equal strain hypothesis in the simple case of no peripheral smear or well resistance, very similar to Barron's approach, was presented already in 1937 (Kjellman, 1948). Kjellman's approach was extended by the author (Hansbo, 1979; 1981) to include the effect of smear and well resistance. In this case, the average degree of consolidation is given by the relation:

$$\overline{U}_{hz} = 1 - \exp\left(-\frac{8c_h t}{\mu D^2}\right) \tag{9}$$

where

$$\mu = \frac{D^2}{D^2 - d_w^2}\left[\ln\left(\frac{D}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4}\right] + \frac{d_s^2}{D^2 - d_w^2}\left(1 - \frac{d_s^2}{4D^2}\right) - \frac{k_h\left(d_s^2 - d_w^2\right)}{k_s\left(D^2 - d_w^2\right)}\left(1 - \frac{d_s^2 + d_w^2}{4D^2}\right) + \frac{k_h \pi z(2l - z)\left[1 - \left(d_w/D\right)^2\right]}{q_w}$$

Omitting terms which are normally of minor significance we find:

$$\mu = \ln\left(\frac{D}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4} + \frac{k_h \pi z(2l - z)}{q_w}$$

The average degree of consolidation $\overline{U}_h$ of the whole layer is obtained by exchanging the value of μ for:

$$\mu_{av} = \ln\left(\frac{D}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4} + \frac{2k_h \pi l^2}{3q_w}$$

Introducing the initial hydraulic head $\Delta h = \overline{u}_0/\gamma_w$ the hydraulic gradient i becomes:

$$i = \frac{\Delta h}{\mu}\left(1 - \overline{U}_{hz}\right)\left(\frac{1}{\rho} - \frac{4\rho}{D^2}\right) \tag{10}$$

The average degree of consolidation achieved by one-dimensional vertical pore water flow (undrained condition) is generally below 50%. Therefore, the total average degree of consolidation for fully penetrating drains, taking into account both undrained (c_v value assumed to be constant) and drained condition can be expressed by the relation:

$$\overline{U}_{av} = 1 - \left(1 - \frac{2}{l}\sqrt{\frac{c_v t}{\pi}}\right)\exp\left(-\frac{8c_h t}{\mu_{av} D^2}\right) \tag{11}$$

where $2l$ = thickness of the clay layer when drained at top and bottom (Fig. 1), corresponding to the length of the drains.

Zeng and Xie (1989) pointed out that the continuity at the drain interface was not satisfied in the solutions given by equations (7) and (9) and presented what they call a 'correct' solution:

$$\overline{U}_{hz} = 1 - \sum_{m=0,1,2,\cdots}^{\infty} \frac{2}{M}\sin\left(\frac{M}{l}z\right)\exp\left(-\frac{8c_h t}{\eta D^2}\right) \tag{12}$$

where $M = \frac{2m+1}{2}\pi$,

$$\eta = \frac{d_w^2}{D^2 - d_w^2}\left\{\begin{aligned}&\frac{D^2}{d_w^2}\left[\ln\left(\frac{D}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4}\right] + \\ &+\left(\frac{d_s}{d_w}\right)^2\left(1 - \frac{k_h}{k_s}\right)\left(1 - \frac{d_s^2}{4D^2}\right) + \frac{k_h}{k_s}\left(1 - \frac{d_w^2}{4D^2}\right)\end{aligned}\right\} + \frac{2k_h \pi l^2\left(D^2 - d_w^2\right)}{M^2 D^2 q_w}$$

With the advances of the finite element and the finite difference methods the consolidation process achieved for any type of loading and drainage condition can be solved theoretically on the basis of given consolidation and drain parameters (e.g. Onoue, 1988; Lo, 1991). Among these can be mentioned the finite element program ILLICON developed at University of Illinois at Urbana-Champaign, USA, which is based on the following basic correlations (Lo, 1991):

$$\left(\frac{\partial e}{\partial \sigma'_v}\right)_t \frac{\partial \sigma'_v}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_{\sigma'_v} = \frac{(1+e_0)^2}{\gamma_w(1+e)}\left[\frac{\partial k_v}{\partial z}\frac{\partial u}{\partial z} + k_v\left(\frac{\partial^2 u}{\partial z^2} - \frac{1}{1+e}\frac{\partial u}{\partial z}\frac{\partial e}{\partial z}\right)\right] +$$

$$+ \frac{1+e}{\gamma_w}\left[\frac{\partial k_h}{\partial \rho}\frac{\partial u}{\partial \rho} + k_h\left(\frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{\partial^2 u}{\partial \rho^2}\right)\right] \tag{13}$$

The excess pore water pressure is given by the relation:

$$\frac{\partial \sigma_v}{\partial t} - \frac{\partial u}{\partial t} = \frac{\partial e/\partial t - (\partial e/\partial t)_{\sigma'_v}}{(\partial e/\partial \sigma'_v)_t} \tag{14}$$

Here $(\partial e/\partial t)_{\sigma'_v}$ = change in void ratio with time at a given effective stress,
$(\partial e/\partial \sigma'_v)_t$ = change in void ratio owing to change in effective stress.

The left-hand term in equation (13) is equivalent to the constitutive relation presented by Taylor & Merchant (1940):

$$\frac{de}{dt} = \left(\frac{\partial e}{\partial \sigma'_v}\right)_t \frac{\partial \sigma'_v}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_{\sigma'_v}$$

The results obtained by different design methods for drains with well resistance are compared in Fig. 4. In the case shown in Fig. 4 the agreement between all the exemplified theories is very good. However, as was shown by Lo (1991), Barron's equal strain solution underestimates the average degree of consolidation when the discharge capacity of the drains is considerably smaller than in Fig. 4. Choosing, for example, q_w = 0.7 m^3/year instead of q_w = 7 m^3/year (all other parameters unchanged), the degrees of consolidation obtained at depth z = 8 m by radial drainage only for the time factors T_h = 0.1, 1.0 and 5.0 become 0.03, 0.23 and 0.73, respectively, according to Barron's solution (equation 7) and 0.05, 0.42 and 0.94, respectively, according to the author's solution (equation 9). The solution given by equation (9) is in good agreement with, for example, the solution given by Lo, equations (13–14).

3.2 Assumptions Based on Non-Validity of Darcy's Law

In the course of consolidation, the permeability in particular will be subjected to gradual reduction. However, case studies and experimental evidence have also shown that the coefficient of consolidation increases with increasing magnitude of the load that produces consolidation. For the determination of the coefficient of consolidation Terzaghi & Peck (1948) therefore recommended that the load

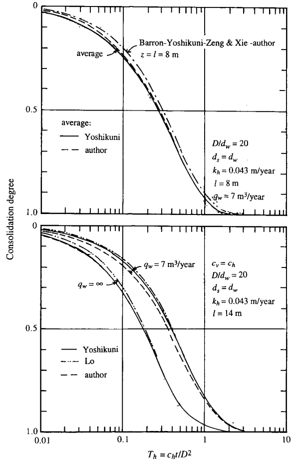

Fig. 4. *Top*: Example showing the effect of well resistance on radial drainage (average and for $z = l$) according to Barron, equation (7), Yoshikuni (L = 1), Zeng & Xie, equation (12), and the author, equation (9). *Bottom*: Example showing the result of combined vertical and radial drainage without and with well resistance according to Yoshikuni (L = 0 and L = 3), Lo (ILLICON) and the author, equation (11).

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Introducing the initial hydraulic head $\Delta h = \bar{u}_0/\gamma_w$ the hydraulic gradient i becomes:

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$$\eta = \frac{d_w^2}{D^2 - d_w^2}\left\{\begin{aligned}&\frac{D^2}{d_w^2}\left[\ln\left(\frac{D}{d_s}\right) + \frac{k_h}{k_s}\ln\left(\frac{d_s}{d_w}\right) - \frac{3}{4}\right] + \\ &+\left(\frac{d_s}{d_w}\right)^2\left(1 - \frac{k_h}{k_s}\right)\left(1 - \frac{d_s^2}{4D^2}\right) + \frac{k_h}{k_s}\left(1 - \frac{d_w^2}{4D^2}\right)\end{aligned}\right\} + \frac{2k_h \pi l^2 \left(D^2 - d_w^2\right)}{M^2 D^2 q_w}$$

With the advances of the finite element and the finite difference methods the consolidation process achieved for any type of loading and drainage condition can be solved theoretically on the basis of given consolidation and drain parameters (e.g. Onoue, 1988; Lo, 1991). Among these can be mentioned the finite element program ILLICON developed at University of Illinois at Urbana-Champaign, USA, which is based on the following basic correlations (Lo, 1991):

$$\left(\frac{\partial e}{\partial \sigma_v'}\right)_t \frac{\partial \sigma_v'}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_{\sigma_v'} = \frac{(1+e_0)^2}{\gamma_w(1+e)}\left[\frac{\partial k_v}{\partial z}\frac{\partial u}{\partial z} + k_v\left(\frac{\partial^2 u}{\partial z^2} - \frac{1}{1+e}\frac{\partial u}{\partial z}\frac{\partial e}{\partial z}\right)\right] + $$
$$+ \frac{1+e}{\gamma_w}\left[\frac{\partial k_h}{\partial \rho}\frac{\partial u}{\partial \rho} + k_h\left(\frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{\partial^2 u}{\partial \rho^2}\right)\right] \tag{13}$$

The excess pore water pressure is given by the relation:

$$\frac{\partial \sigma_v}{\partial t} - \frac{\partial u}{\partial t} = \frac{\partial e/\partial t - (\partial e/\partial t)_{\sigma_v'}}{(\partial e/\partial \sigma_v')_t} \tag{14}$$

Here $(\partial e/\partial t)_{\sigma_v'}$ = change in void ratio with time at a given effective stress,
$(\partial e/\partial \sigma_v')_t$ = change in void ratio owing to change in effective stress.

The left-hand term in equation (13) is equivalent to the constitutive relation presented by Taylor & Merchant (1940):

$$\frac{de}{dt} = \left(\frac{\partial e}{\partial \sigma_v'}\right)_t \frac{\partial \sigma_v'}{\partial t} + \left(\frac{\partial e}{\partial t}\right)_{\sigma_v'}$$

The results obtained by different design methods for drains with well resistance are compared in Fig. 4. In the case shown in Fig. 4 the agreement between all the exemplified theories is very good. However, as was shown by Lo (1991), Barron's equal strain solution underestimates the average degree of consolidation when the discharge capacity of the drains is considerably smaller than in Fig. 4. Choosing, for example, q_w = 0.7 m³/year instead of q_w = 7 m³/year (all other parameters unchanged), the degrees of consolidation obtained at depth z = 8 m by radial drainage only for the time factors T_h = 0.1, 1.0 and 5.0 become 0.03, 0.23 and 0.73, respectively, according to Barron's solution (equation 7) and 0.05, 0.42 and 0.94, respectively, according to the author's solution (equation 9). The solution given by equation (9) is in good agreement with, for example, the solution given by Lo, equations (13–14).

3.2 Assumptions Based on Non-Validity of Darcy's Law

In the course of consolidation, the permeability in particular will be subjected to gradual reduction. However, case studies and experimental evidence have also shown that the coefficient of consolidation increases with increasing magnitude of the load that produces consolidation. For the determination of the coefficient of consolidation Terzaghi & Peck (1948) therefore recommended that the load

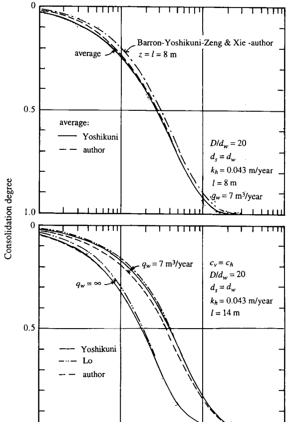

Fig. 4. *Top*: Example showing the effect of well resistance on radial drainage (average and for $z = l$) according to Barron, equation (7), Yoshikuni (L = 1), Zeng & Xie, equation (12), and the author, equation (9). *Bottom*: Example showing the result of combined vertical and radial drainage without and with well resistance according to Yoshikuni (L = 0 and L = 3), Lo (ILLICON) and the author, equation (11).

increment "applied to the sample after a pressure equal to the overburden pressure has been reached should be of the same order of magnitude as the load per unit area of the base of the structure". A possible explanation for this phenomenon can be an exponential correlation between pore water flow and hydraulic gradient.

Results of permeability tests on clay samples presented by different researchers (e.g. Silfverberg, 1947; Hansbo, 1960; Miller & Low, 1963; Dubin & Moulin, 1986; Zou, 1996) have indicated that the pore water flow v caused by a hydraulic gradient i may deviate from Darcy's law $v = ki$ where k is the coefficient of permeability. Silfverberg and Miller & Low drew the conclusion that there is a threshold gradient i_0 below which no flow will take place, yielding $v = k(i - i_0)$, while the author (Hansbo, 1960) proposed the following relation, Fig. 5:

$$v = \kappa i^n \text{ when } i \le i_l \tag{15}$$

$$v = \kappa n i_l^{n-1}(i - i_0) \text{ when } i \ge i_l \tag{16}$$

In the author's opinion, $i_l = i_0 n/(n-1)$ represents the gradient required to overcome the maximum binding energy of mobile pore water [the physical background to non-linear conductivity behaviour is discussed in detail by Hansbo (1960)].

The relations (15)–(16) proposed by the author were also chosen by Dubin and Moulin (1986) in the analysis of Terzaghi's one-dimensional consolidation theory denoting $\kappa = ak$ where $a = n^{-1} i_l^{1-n}$ and $k = v/(i - i_0)$. The values of i_l have been found to vary from 4–10 (Hansbo, 1960) and 8–35 (Dubin & Moulin, 1986). Using the non-linear flow law given by equation (15) the consolidation equation, taking both smear and well resistance into account, can be written:

$$\overline{U}_{hz} = 1 - \left[1 + \frac{\lambda t}{\alpha D^2}\left(\frac{\Delta h}{D}\right)^{n-1}\right]^{-\frac{1}{n-1}} \tag{17}$$

where $\Delta h = \overline{u}_0/\gamma_w$ = the average increase in piezometric head caused by the placement of the load,

$\lambda = \kappa_h M/\gamma_w$ = the coefficient of consolidation,

$$\alpha = \frac{n^{2n}\beta^n}{4(n-1)^{n+1}}$$

with

$$\beta = \frac{1}{3n-1} - \frac{n-1}{n(3n-1)(5n-1)} - \frac{(n-1)^2}{2n^2(5n-1)(7n-1)} +$$
$$+\frac{1}{2n}\left[\left(\frac{\kappa_h}{\kappa_s}-1\right)\left(\frac{D}{d_s}\right)^{(1/n-1)} - \frac{\kappa_h}{\kappa_s}\left(\frac{D}{d_w}\right)^{(1/n-1)}\right] +$$
$$+\frac{(1-1/n)(d_w/D)^{(1-1/n)}\left(1-d_w^2/D^2\right)^{1/n}\kappa_h \pi z(2l-z)}{2q_w}$$

The average degree of consolidation $\overline{U}_h$ for the whole layer is obtained by exchanging the last term in the β expression for:

$$\frac{(1-1/n)(d_w/D)^{(1-1/n)}\left(1-d_w^2/D^2\right)^{1/n} k_h \pi l^2}{3q_w}$$

When the exponent $n \to 1$, equation (17) yields the same result as equation (9) assuming $\lambda = c_h$ and $\kappa_h = k_h$ and $\kappa_s = k_s$. Thus equation (17) is generally applicable and can, therefore, replace equation (9).

The hydraulic gradient i becomes equal to:

$$i = \frac{\Delta h}{D}\left(1-\overline{U}_{hz}\right)\left[\frac{1}{4\alpha(n-1)}\left(\frac{D}{2\rho} - \frac{2\rho}{D}\right)\right]^{1/n} \tag{18}$$

The best agreement between theory and observations has been obtained for n = 1.5 (Hansbo, 1960; 1997a,b) which yields:

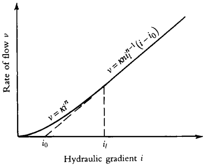

Fig. 5. Hypothetical deviation from Darcy's law based on experimental evidence from results of permeability tests (Hansbo, 1960).

$$\overline{U}_{hz} = 1 - \left(1 + \frac{\lambda t}{\alpha D^2}\sqrt{\frac{\Delta h}{D}}\right)^{-2} \tag{19}$$

where $\alpha = 4.77\beta\sqrt{\beta}$

and, omitting terms of minor significance,

$$\beta = 0.270 + \frac{1}{3}\left[\left(\frac{\kappa_h}{\kappa_s}-1\right)\left(\frac{d_w}{d_s}\right)^{1/3} - \frac{\kappa_h (d_w/D)^{1/3}}{\kappa_s} + \frac{(d_w/D)^{1/3}\kappa_h \pi z(2l-z)}{2q_w}\right]$$

The average degree of consolidation $\overline{U}_h$ for the whole layer is obtained by exchanging β for:

$$\beta_{\text{av.}} = 0.270 + \frac{1}{3}\left[\left(\frac{\kappa_h}{\kappa_s}-1\right)\left(\frac{d_w}{d_s}\right)^{1/3} - \frac{\kappa_h (d_w/D)^{1/3}}{\kappa_s} + \frac{(d_w/D)^{1/3}\kappa_h \pi l^2}{3q_w}\right]$$

Inserting β_{av}, the total average degree of consolidation, taking into account both undrained and drained conditions, can be expressed by the approximate relation (approximate since the contribution by one-dimensional vertical consolidation is based on validity of Darcy's law and thus inconsistent with the λ theory):

$$\overline{U}_{\text{av}} = 1 - \left(1 - \frac{2}{l}\sqrt{\frac{c_v t}{\pi}}\right)\left(1 + \frac{\lambda t}{\alpha_{\text{av}} D^2}\sqrt{\frac{\Delta h}{D}}\right)^{-2} \tag{20}$$

where $2l$ = thickness of the clay layer when drained at top and bottom (Fig. 1), corresponding to the length of the drains.

The influence of various magnitudes of well resistance on the results obtained according to equation (19) is exemplified in Fig. 6.

In field conditions the hydraulic gradient i in most cases is very small in comparison with laboratory conditions. Choosing as an example a case with $n = 1.5$, $\Delta h = 5$ m, $D = 1.05$ m, $d_w = 0.066$ m and $d_s = 0.15$ m (no well resistance), equation (14) in the initial state $\left(\overline{U}_{hz} = 0\right)$ yields $i < 10$ in 85 % of the total drained cylinder. At an average degree of consolidation of 50 %, $i < 10$ in 98 % of the total drained cylinder. If Δh = 8 m the corresponding figures become 65 % and 91 %, respectively. Thus, if we have to deal with non-Darcian flow with values of i_l about 10, equation (19) can replace equation (9) in most cases, except when the quotient $\Delta h/D$ is very large. In the latter case, one had better apply equation (9).

It is interesting to note that Schiffman's approach with a gradual decrease of the coefficient of permeability in the course of consolidation has an effect on the consolidation rate similar to that based on non-Darcian flow. However, Schiffman's

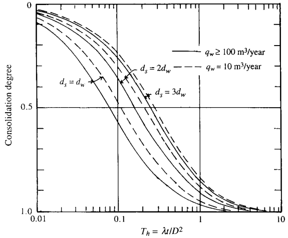

Fig. 6. Example showing the effect of smear and well resistance according to equation (19) for $D/d_w = 20$, $z = l = 10$ m, $\kappa_h = 0.03$ m/year, $\kappa_h/\kappa_s = 4$ and $\Delta h/D = 2$.

concept is difficult to apply in practical design and can also be questioned from a physical point of view. The assumption of a linear correlation between permeability coefficient and excess pore water pressure is not verified. Moreover, the coefficient of consolidation determined by oedometer tests tends to increase with increasing effective pressure which contradicts the concept put forward by Schiffman.

4. CHOICE OF PARAMETERS

4.1 Equivalent Diameter of Band drains

The first type of band drains, the so-called cardboard wick, which was invented and introduced on the market by the Swedish Geotechnical Institute, was assumed by Kjellman (1948) to have an equivalent diameter of 50 mm. The author (Hansbo, 1979) showed that the process of consolidation for a circular drain and a band drain is very nearly the same if the band drain is assumed to have an equivalent diameter:

$$d_w = 2(b+t)/\pi \tag{21}$$

where b = the width of the drain and t = the thickness of the drain.

Table 1. Characteristics of various band drains

Drain make	Core width mm	Core thickness mm	Filter sleeve	$d_{weq.}$ mm
Alidrain	100	6	yes	67
Amerdrain	92	10	yes	65
Bando Chemical	96	2.9	yes	63
Cardboard Wick	100	3	no	66
Castleboard	94 ± 2	2.6 ± 0.5	yes	62
Colbond CX 1000	100	5	yes	67
Desol	95	2	no	62
Fibredrain	80–100	8–10	yes	63
Flodrain	95	4	yes	63
Geodrain, L-type	95.8 ± 2.0	3.4 ± 0.5	yes	63
Geodrain, M-type	95.8 ± 2.0	4.2 ± 0.5	yes	64
Mebradrain	100	3–4	yes	66
OV-drain	103	2.5	no	67
PVC	100	1.6 ± 0.2	no	65
Tafnel	102	6.9	no	69

According to Atkinson and Eldred (1981), the diameter given by Eq. (21) should be reduced for the effect of convergence of flow lines towards the corners of the band drain and propose:

$$d_w = (b+t)/2 \tag{22}$$

The magnitude D/d_w in the latter case will increase by about 27%. Which of the relations (21) or (22) to be used is an open question, but the difference in result between the two is insignificant in comparison with the influence on the result exerted by the choice of other consolidation parameters to be applied in the design.

The equivalent diameter of various makes of band drains according to equation (21) is shown in Table 1.

4.2 The Zone of Smear

The effect on the consolidation parameters of disturbance caused by the installation of drains, expressed in the terms of zone of smear, depends very much on the method of drain installation, the size and the shape of the mandrel, and the soil structure (Sing & Hattab, 1979; Bergado *et al.*, 1993). Two problems exist: to find the correct diameter value d_s of the zone of smear and to evaluate the effect of smear on the permeability.

The first problem has been the subject of a number of investigations in connection with pile installations which remind of installations by means of closed-ended circular mandrels. These investigations indicate that the diameter d_s of the zone of smear can be assumed equal to 1.5–3 times the diameter d_m of a circular mandrel. Investigations on a laboratory scale (Bergado *et al.*, 1992) indicate that d_s can be put equal to $2d_m$. If the mandrel is non-circular, the diameter d_s then should yield an area corresponding to about 4 times the cross-sectional area of the mandrel, or in the case of band drains, that of the folded anchor utilised for anchoring the drains in the soil. In many cases this cross-sectional area is typically about 7000 mm² in size which yields $d_s \approx 0.19$ m.

Several authors have treated the other problem, the choice of permeability in the zone of smear. Of course, the permeability in the zone of smear will vary from a minimum nearest to the drain to a maximum at the outer border of the zone. The most conservative solution to the problem is to assume that horizontal layers in the undisturbed soil are turned vertical in the zone of smear, resulting in the quotient k_h/k_s being equal to the quotient $k_h/k_v = c_h/c_v$ (see also Bergado *et al.*, 1992).

Onoue *et al.* (1991) and Madhav *et al.* (1993) divide the zone of smear into two subzones: an inner, highly disturbed zone, and an outer transition zone in which the disturbance decreases with increasing distance from the drain. Madhav *et al.* conclude that the author's solution based on axi-symmetric smear conditions and the assumption of only one smear zone is "reasonably accurate for all practical purposes". Chai *et al.* (1997) use a linear variation of the permeability in the zone of smear on one hand and a bilinear variation on the other and conclude that the assumption of one single average value of permeability will underevaluate the effect of smear. Hird and Moseley (1998) conclude, on the basis of laboratory tests, that the ratio k_h/k_s for layered soil, assuming $d_s = 2d_w$, can be much larger than that mentioned in the literature but that the assumption of $k_s = k_v$ in these cases is much too severe.

4.3 Requirements on Band Drains

Most of the band drains are made up of a central core with longitudinal channels surrounded by a filter of synthetic material. A first important requirement on these materials is that they must be strong enough to resist the tension and the wear and tear which takes place during drain installation.

Much concern has been devoted to filter criteria. Among the problems mentioned the risk of siltation and the strength of the filters have to be taken into account. The risk of blinding owing to too low a permeability of the filter has almost negligible effect on the consolidation behaviour. The filter and the low-permeable cake of soil particles which may be formed outside the filter and cause so-called blinding will have a fairly small thickness. The consequence of this blinding is easily recognised if the filter is considered as a zone of smear. Assuming, for example, that the filter/filter cake has a thickness of as much as 2 mm (corresponding to $d_s = d_w + 0.004$ m) and that its permeability becomes only 20% of the permeability of the surrounding soil ($k_s/k_h = 0.2$) the average degree of consolidation, using band drains with an equivalent diameter $d_w = 0.066$ m, will differ from the ideal case by a maximum of only 2–3 %, a negligible difference in result.

The filter material has also been considered important. When the first modern prefabricated band drain, the Geodrain, was introduced on the market the filter was made of specially prepared paper material. Although the effectiveness of these drains was demonstrated by the results of a large number of drain installations the use of paper as filter material was questioned. The main reason for questioning the use of paper was the risk of filter deterioration caused by fungi or bacteria. This risk has proved by full-scale experiments to be overstated, Fig. 7. Moreover, there are cases where clogging of the drains would be desirable after that full consolidation under the design load has been attained, which generally requires a preconsolidation time of about one year.

4.4 Well Resistance

Because drains nowadays are frequently installed to great depths well resistance has become a matter of increasing interest. This is understandable since well resistance in such cases can cause a serious delay in the consolidation process. There are several reasons why the discharge capacity of a drain may become low: siltation of the channels in the core of band drains or of the sand in sand drains; unsatisfactory drain makes with too low a discharge capacity; necking of drains; etc. Back-calculated values of discharge capacity of drains under field conditions have been reported to be quite low for certain makes of band drains without filter (Hansbo, 1986; Chai *et al.*, 1996). However, most of the band drains marketed today have high enough a discharge capacity ($q_w > 150$ m^3/year) to become negligible in the design (*cf.* Hansbo, 1986; 1994). Moreover, the influence of well resistance decreases with increasing time of consolidation.

Well resistance may be important in the case of small-diameter sand drains. For example, using medium to coarse sand as material in the drains, the permeability can be expected to be $k_w \approx 3000$ m/year (10^{-4} m/sec.). For sandwicks, 0.05 m in diameter, and fabri pack drains, 0.12 m in diameter, this yields $q_w = 6$ m^3/year and 34 m^3/year respectively. In those cases where well resistance has to be considered

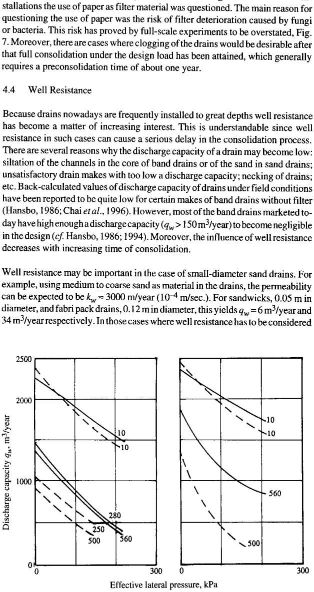

Fig. 7. Influence on discharge capacity of filter deterioration (Koda *et al.*, 1986). Tests on band drains (type Geodrain) with filter sleeves of paper (broken lines) and synthetic material (full lines) which were pulled out of peat (to the left) and gyttja (to the right) after different lengths of time after installation (number of days given in figure).

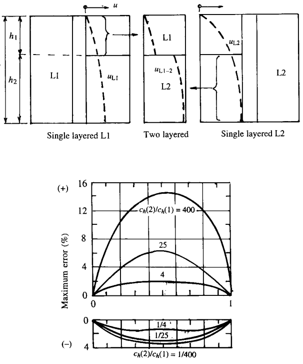

Fig. 8. Consolidation of anisotropic soil by vertical drains with well resistance exemplified for a two-layered soil deposit with different characteristics. Simplified solution (top) and consequential error according to Onoue (1988).

the magnitude of the permeability k_h (κ_h) has an important influence on the result obtained. Choosing as an example a discharge capacity $q_w = 10$ m^3/year and the time factor $T_h = \lambda t/D^2 = 0.1$, but changing κ_h from 0.03, see Fig. 4, to 0.08, the average degree of consolidation $\overline{U}_{hz}$ for $d_s = 2d_w$ and $z = l = 10$ decreases from 0.28 to 0.20.

The effect of well resistance according to equations (7) and (9) and equations (17) and (19) is calculated on the assumption that we have to deal with homogeneous soil conditions (k_h and κ_h assumed constant). However, if the soil consists of layers with different characteristics, this can be taken into account in a simple way as suggested by Onoue (1988). The consolidation process is calculated on the assumption that the whole soil profile is homogeneous and has the consolidation properties of each of the respective layers. The distribution of excess pore pressure in the respective layer is extracted and plotted as shown in Fig. 8.

4.5 Correlation between λ and c_h

The ratio of λ to c_h will depend on the hydraulic gradient prevailing in the horizontal direction during the consolidation process. This value can be estimated on the basis of the expression for i, given by equation (10), or more general by equation (18). Since the parameters M and γ_w are independent of the flow conditions, we have $c_h/\lambda = k_h/\kappa_h$. Equalising the areas created below the flow *vs.* gradient curves in the two cases Darcian and non-Darcian flow, we find the correlation:

$$k_h \approx \frac{2i^{n-1}}{n+1}\kappa_h \text{ when } i \le i_l \tag{23}$$

and

$$k_h \approx \frac{2}{i^2}\left[\frac{i_l^{n+1}}{n+1} + n i_l^{n-1}(i - i_l)\left(\frac{i - i_l}{2} + \frac{i_l}{n}\right)\right]\kappa_h \text{ when } i \ge i_l \tag{24}$$

Assuming, for example, that the maximum gradients reached during the consolidation process are respectively 2, 5, 15, 25 and 75 and that the exponent n = 1.5 and the limiting gradient i_l = 8, we find in due order $\lambda/c_h = \kappa_h/k_h \approx$ 0.88, 0.56, 0.34, 0.29 and 0.25. Thus, the higher the value of Δh and the smaller the drain spacing, the lower the ratio of λ to c_h.

Provided n = 1.5 and $i_{max} \le 2.5 i_l$, the correlation between c_h and λ can be determined approximately from the relation:

$$\frac{c_h}{\lambda} = 0.8\sqrt{i_{max}} \tag{25}$$

where $i_{max} \approx \frac{\Delta h}{D}\left[\frac{1}{2\alpha}\left(\frac{D}{d_s} - \frac{d_s}{D}\right)\right]^{2/3}$

If $i_{max} > 2.5 i_l$, where $1.5\sqrt{i_l} > 1$, this correlation should be replaced by:

$$\frac{c_h}{\lambda} = \frac{2}{i_{max}^2}\left[\frac{i_l^{2.5}}{2.5} + 1.5\sqrt{i_l}\,(i_{max} - i_l)\left(\frac{i_{max}}{2} + \frac{i_l}{6}\right)\right] \tag{26}$$

In the latter case the result is very sensitive to the value of i_l selected.

5. SETTLEMENT ANALYSIS

Among the main problems to resolve remain the predetermination of the primary consolidation settlement and the influence of secondary consolidation. This predetermination is generally based on the results of oedometer tests, which may give quite misleading information about the deformation properties of the soil owing to sample disturbance. Empirical correlations are also utilised as means of establishing the deformation characteristics. A misinterpretation of the preconsolidation pressure or of the consolidation parameters (compression modulus, coefficient of consolidation or permeability) may give a completely wrong picture of the final settlement and the consolidation rate to be expected.

5.1 Determination of the Preconsolidation Pressure

The preconsolidation pressure σ_c' is usually determined according to the well-known Casagrande procedure from the shape of the oedometer curve presented in the logσ'/ε (or logσ'/e) diagram. This method, however, may give quite a false picture of the preconsolidation pressure in the case of disturbed soil samples. Therefore, it is always necessary to check the shape of the oedometer curve in linear scales σ'/ε (or σ'/e). As demonstrated in Fig. 9, the use of logσ'/ε diagrams may give the impression of the existence of a preconsolidation pressure where it cannot be noticed because of sample disturbance.

The preconsolidation pressure will be affected by the disturbance caused by drain installations. Lightly overconsolidated clay may turn into normally consolidated clay after that the drains have been installed. This has to be taken into account when calculating the settlement to be expected under an embankment or an area provided with vertical drains. Thus, a certain underestimation of the preconsolidation pressure in this case may be justified. On the other hand, one has to take care not to mislead the building proprietor to install vertical drains where drains are not required.

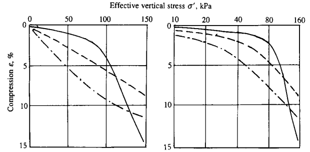

Fig. 9. Results of oedometer tests reported by consultants in semi-log diagrams. Preconsolidation pressures were determined according to the Casagrande method and found to exist in all cases. The lin-lin diagrams show that a preconsolidation pressure can be derived only in one case.

5.2 Primary Settlement

In the analysis of primary settlement the traditional approach is to use the virgin compression ratio $CR = C_c/(1+e_0)$, which represents the relative compression $\varepsilon = \Delta h/h_0$ achieved along the virgin oedometer curve (C_c = the compression index; e_0 = the initial void ratio; h_0 = the initial height of the oedometer sample). The compression ε is then obtained by the relation:

$$\varepsilon = RR\lg\left(\frac{\sigma_c'}{\sigma_0'}\right) + CR\lg\left(\frac{\sigma' - \sigma_c'}{\sigma_c'}\right) \tag{27}$$

where RR = recompression ratio (overconsolidated state),
σ'_0 = effective overburden pressure,
σ'_c = preconsolidation pressure,
σ' = effective vertical stress at the end of the primary consolidation period.

However, the simplest way of determining the primary settlement is usually to make a direct analysis of the compression achieved along the oedometer curves for the stress increments in question. Regard ought to be paid to the fact that settlement leads to a gradual reduction of the load in accordance with Archimedes's principle.

In the analysis one has to take into account the influence on the compression characteristics of the disturbance taking place during drain installations. These disturbance effects are not compensated by the disturbance due to sampling. Thus, sample disturbance results in a lower value of the compression index, whereas disturbance of the soil in nature results in a higher value of the compression index.

Owing to the uncertainties involved in the calculation of the total primary consolidation settlement, test areas are recommended wherever possible. In this case the primary consolidation settlement can be determined according to Asaoka (1978). Asaoka's method is based on the following procedure. The settlement observed at different equal time intervals Δt is plotted in a diagram with s_{i-1} as ordinate and s_i as abscissa where indices i–1 and i refer to times $t - \Delta t$ and t. The primary consolidation settlement is obtained when $s_{i-1} \to s_i$.

5.3 Secondary Settlement

Secondary consolidation refers to the change in void ratio (relative compression) taking place with time at a given effective vertical stress, cf. equation (13). Thus, secondary consolidation has to be taken into account during the whole of the consolidation process. In the case of vertical drainage, however, the primary consolidation period is generally short enough for the influence of secondary consolidation to be ignored meanwhile. Secondary consolidation settlement can therefore be analysed in the traditional way, i.e. starting at the end of the primary consolidation period.

The secondary compression during time Δt can be obtained by the relation:

$$\varepsilon = \alpha_s \lg\left[\left(t_p + \Delta t\right)/t_p\right] \tag{28}$$

where $\alpha_s = C_\alpha/(1+e_0)$,
C_α = secondary compression index

The secondary compression ratio α_s represents the inclination of the rectilinear tail of the oedometer curve in the lgt/ε diagram (Buisman, 1936). This requires that the oedometer test be carried out by means of a stepwise load increase. Nowadays, however, oedometer tests are often performed as CRS (constant rate of strain) tests which makes impossible a direct judgement of the secondary compression indexes. Therefore, one often has to rely on half-empirical correlations. According to Mesri and Godlewski (1977) the most typical values of C_α/C_c (in other words, the ratio of secondary compression ratio to primary compression ratio) are 0.04 ± 0.01 for inorganic soft clays and 0.05 ± 0.1 for organic soft clays.

6. MONITORING OF VERTICAL DRAIN PROJECTS

Monitoring of vertical drain projects is more or less a must since the consolidation characteristics determined by oedometer tests may be misleading. An early follow-up of the results obtained will form the basis for a correct estimate of the result to be expected, so-called active design.

The monitoring systems utilised for control of vertical drain projects usually consist of vertical settlement meters of various types and of piezometers placed at different depths in the soil. In the case of pilot tests the size of the test area is often limited in relation to the thickness of the soil layer subjected to consolidation. Therefore, in such cases the influence on the vertical settlement of horizontal displacements has to be taken into account. This purpose is usually achieved by the installation of inclinometers along the border of the test area.

Considering the derivation of the consolidation theory, the follow-up of the consolidation process nearest at hand is to check the course of excess pore pressure dissipation. However, experience shows that the interpretation of the consolidation process on the basis of pore pressure measurements may be quite intricate. The main problem in the case of vertically drained areas consists in uncertainty about the exact position of the filter tip of the piezometer in relation to the surrounding drains. Therefore, the observations can give a misleading conception of the average excess pore water pressure dissipation. To find the average degree of consolidation on the basis of pore pressure observations, the piezometer tip should be placed about halfway between the outer border of the drained cylinder and the drain (*cf.* Fig. 2). However, in practice the ambition is generally to have it placed at the outer border of the drained cylinder (halfway between the drains). Other difficulties arise from the fact that the piezometer tip, owing to frictional forces against the piezometer tube by settlement of overlying soil layers, may be penetrating the underlying soil, thereby creating additional excess pore pressure. Phenomena such as influence of pore gas, erroneous pore pressure readings, collapse of soil structure, structural viscosity, secondary consolidation, and the fact that the ground water level may not revert to its original position, may also contribute to discrepancies observed between the consolidation degree based on settlement and that based on pore pressure observations. One must also bear in mind that the drain installation in itself causes excess pore water pressure which may extend even far outside the drained area (*cf.* Hansbo, 1960).

The aim of preloading in combination with vertical drainage is usually to eliminate unacceptable settlement under future loading conditions. The preconsolidation pressure in the soil has to be increased up to, or preferably above, the effective stress level induced by the future load. Settlement observations of the soil surface may be strongly influenced by vertical consolidation U_v and thus lead to the impression that the acceleration of the consolidation process caused by the drains is faster than in reality. In active design, this can be checked theoretically by inserting the values of c_v and $c_h(\lambda)$, found by trial and error, that yield acceptable agreement with the course of surface settlement. Then the degree of consolidation obtained by the aid of the drains can be checked by inserting $c_v = 0$ and the value of $c_h(\lambda)$ found. However, if there are layers with more unfavourable consolidation characteristics than on the whole, these layers will be decisive.

7. CASE RECORDS

It is interesting to check by case records whether the theory based on non-Darcian flow gives results in better agreement with real behaviour than the theory based on validity of Darcy's law. As already mentioned, the author in the study of the full-scale tests at Skå-Edeby (Hansbo, 1960) found that the best agreement between theory and practice was obtained by assuming non-Darcian flow with the exponent $n = 1.5$–1.6. The findings then presented, assuming $n = 1.5$, i.e. the flow law $v = \kappa i\sqrt{i}$, have been confirmed in later studies (Robertson *et al.*, 1988; Hansbo, 1994; Hansbo, 1997a; 1997b).

In a case where the monitoring system is based on settlement observations of the soil surface (which is most common) the settlement observed *vs.* total settlement refers to the average degree of consolidation. Assuming that Darcy's law is valid, the settlement due to one-dimensional vertical consolidation can be obtained utilising the diagrams of average one-dimensional consolidation *vs.* time factor T_v found in most textbooks on soil mechanics. If, on the other hand, the stress increase varies non-linearly with depth below the ground surface, or if a limited layer at a certain depth is considered, then the effect of one-dimensional vertical consolidation can be calculated by finite difference methods as suggested by Helenelund (see Hansbo, 1994). The effect of one-dimensional consolidation assuming non-Darcian flow can be studied in the paper presented by Dubin and Moulin (1986). The difference in result as compared to Terzaghi's solution will be most pronounced at the end of the consolidation process. Therefore, and in consideration of all the uncertainties involved in the choice of consolidation parameters, Terzaghi's solution can be applied when judging the influence of one-dimensional vertical outflow of water. This concept forms the basis of equations (11) and (20) submitted by the author. The best thing, of course, is to study the settlement of a dummy area without drains and with similar loading conditions.

7.1 The Skå-Edeby Test Field

The test field arranged at Skå-Edeby, situated some 25 km west of Stockholm, is one of the oldest and best documented test fields throughout the world. It was established by the Swedish Government in 1957 for the purpose of examining the effectiveness of vertical sand drains in a then planned soil improvement project for a new International Airport. (For full details about the test field, see Hansbo, 1960).

The soil conditions in the test field can be summarised as follows. Below a 1.5 m thick dry crust, the soil consists of normally consolidated, highly plastic clay to a depth of 9–15 m (average about 12 m). From the results of oedometer tests the following consolidation characteristics were found: coefficients of consolidation $c_v = 0.17$ m²/year (standard deviation 0.03 m²/year) and $c_h = 0.7$ m²/year (only one test); compression ratio $CR = 0.37$ (standard deviation 0.06). A corrected interpretation of the results of the oedometer tests with regard to disturbance of the soil samples resulted in $CR = 0.52$ (standard deviation 0.18).

Comparisons between theoretical and measured consolidation rates earlier presented by the author (Hansbo, 1997b), valid for Test Areas I (sand drains; 0.9 m spacing), II and III (sand drains; 1.5 m spacing; different loading conditions) and V (band drains, type Geodrain with paper filter; 0.9 m spacing; load doubled after 3 years of consolidation) at Skå-Edeby, have shown that the consolidation theory based on non-Darcian flow gives much better agreement with observations than the consolidation theory based on Darcian flow.

In this paper the results obtained in Test Areas I and II will be examined. Test Area I, 70 m in diameter, is divided into three equal sectors with drain spacing 0.9 m, 1.5 m and 2.2 m. Test Area II, 35 m in diameter, has a drain spacing of 1.5 m. The depth of the clay layer in these two cases is around 12 m. (The reason why Test Area II is chosen instead of the sector in Test Area I with 1.5 m drain spacing is that the thickness of the clay layer in this sector is considerably smaller than in the sector with 2.2 m drain spacing). The installation of the sand drains was carried out in equilateral triangular pattern by means of a closed-ended mandrel, 0.18 m in diameter. The results obtained in a dummy area without drains, Test Area IV, 35 m in diameter and the same loading condition and clay thickness as in Test Areas I and II, are used for the analysis of the settlement contribution owing to one-dimensional consolidation. The load placed on the test areas consisted of 1.5 m of sand and gravel, corresponding to a load of 27 kN/m². The analysis is based on the following assumptions: $d_w = 0.18$ m, $d_s = 2d_w$ and $k_h/k_s = k_h/k_v$ $(= \kappa_h/\kappa_s) = 4$. Owing to submergence of the fill during the course of settlement, the load, in kN/m², will be reduced successively by about $7s$, where s = settlement, in m, of the soil surface.

The primary compression to be expected on the basis of the compression characteristics of the clay layer between depths 2.5 and 7.5 m was estimated at 0.5 m in the sector with 2.2 m drain spacing in Test Area I and at 0.47–0.54 m in Test Area IV (Hansbo, 1960). A follow-up of the course of settlement according to Asaoka (1978), results in the following relation (see Section 5.2): $s_i = 0.1453 + 0.7882s_{i-1}$ in Test Area I, which yields the primary compression $s_p = 0.69$ m, and $s_i = 0.086 + 0.811s_{i-1}$ in Test Area IV, which yields the primary compression $s_p = 0.46$ m. The total primary settlement in Test Area I and IV was estimated at

1.3 m (Hansbo, 1960) while Asaoka's method yields 1.26 m in Test Area I and 1.12 m in Test Area IV.

The influence of undrained condition on the consolidation process, obtained according to equation (4) from the measurements carried out in the undrained Test Area IV, is shown in Fig.10, top. The result obtained when the influence of undrained condition is neglected is shown in Fig. 10, bottom.

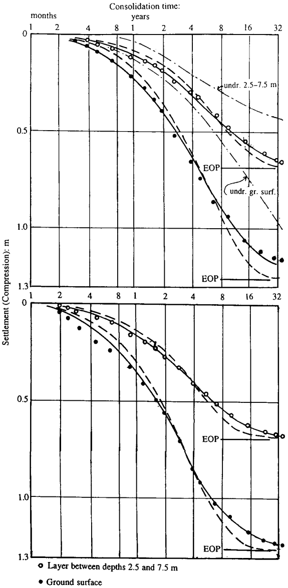

Fig. 10. Results of settlement observations at Skå-Edeby, Sweden. Test Area I: 2.2 m drain spacing (D = 2.31 m). *Top*: observations corrected with regard to influence of undrained contribution to settlement (compression) according to equation (4). *Bottom*: uncorrected observations. EOP = end of primary consolidation settlement (compression) estimated according to Asaoka's method. Full lines: analytical results according to equation (19). Broken lines: analytical results according to equation (9). Horizontal displacements negligible (*cf.* Hansbo, 1960). Settlement corrected with regard to immediate elastic settlement.

The theoretical compression according to equations (9) and (19) of the 5 m thick layer shown in Fig. 10 is obtained for λ = 0.25 m^2/year and c_h = 0.35 m^2/year (undrained condition considered) and for λ = 0.4 m^2/year and c_h =0.6 m^2/year (undrained condition neglected). The surface settlement curves are obtained for λ = 0.3 m^2/year and c_h = 0.45 m^2/year (undrained condition considered) and for λ = 0.5 m^2/year and c_h =0.75 m^2/year (undrained condition neglected). As can be seen, the difference in result obtained when neglecting the effect of undrained condition is only reflected through the values of coefficient of consolidation found by trial and error to give the best fit to field data. The results obtained from the settlement observations in the undrained area show that the drainage conditions are considerably more favourable than would be the case according to one-dimensional consolidation theory based on the total thickness of the clay layer. Most probably, pervious layers in the clay deposit and fissures in the dry crust contribute to shortening the drainage paths. As was shown by the author (Hansbo, 1960), escape of pore water in a horizontal outward direction also has an appreciable influence on the settlements observed in this case.

An alternative method of determining the course of settlement of the ground surface can be made according to equation (11). Inserting the values l = 6 m (total thickness of the clay layer, drained at top and bottom, is about 12 m), c_v = 0.16 m^2/year and $c_h = 4c_v$ = 0.64 m^2/year, the settlement curve shown in Fig. 11 is obtained. The agreement between theory and observations according to the classical theory has now improved but is not as good as the agreement obtained on the basis of equation (20), inserting c_v = 0.16 m^2/year and λ = 0.45 m^2/year. In Fig. 11, a similar comparison is made between the settlement curves obtained according to equations (11) and (20) and observations of the ground surface settlement in Test Area II. Inserting in this case l = 6 m, c_v = 0.22 m^2/year, $c_h = 4c_v$ = 0.88 m^2/year and λ = 0.58 m^2/year, a much better agreement is obtained on the assumption of non-Darcian flow, equation (20), than on the assumption of Darcian flow, equation (11).

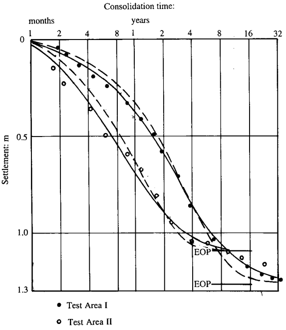

Fig. 11. Settlement of ground surface obtained at Skå-Edeby in Test Area I: 2.2 m drain spacing (D = 2.31 m) and in Test Area II: 1.5 m drain spacing (D = 1.58 m) EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (20). Broken lines: analytical results according to equation (11).

According to equation (25) the values $c_h = 0.64$ m^2/year and $\lambda = 0.45$m^2/year for Test Area I with 2.2 m drain spacing and the values $c_h = 0.88$ m^2/year and $\lambda = 0.58$ m^2/year for Test Area II with 1.5 m drain spacing agree with the observed excess pore pressure value ($\Delta h = 2.4$ m) in the undrained Test Area IV at the beginning of the consolidation process (Skempton's pore pressure coefficients $B = 1$ and $A = 0.77$, *cf.* Hansbo, 1960), yielding $i_{max} = 3.2$ in Test Area I and $i_{max} = 3.6$ in Test Area II. According to equation (26) this corresponds to $i_l = 2.2$.

In practice, the influence of one-dimensional vertical consolidation exerted on the consolidation process at normal drain spacing and thickness of the drained layer is unimportant for the evaluation of the drainage project. It may have an important influence, however, if the drain spacing relatively speaking is large as compared to the thickness of the drained layer.

7.2 The Bangkok Test Field

In connection with the planning of a new international airfield in Bangkok, Thailand, three test areas were arranged in order to form a basis for the design of soil improvement by preloading in combination with vertical drains. The results of the settlement observations in two of these test areas, TS 1 and TS 3, (Hansbo, 1997b) showed a better agreement with equation (19) than with equation (9). In this paper, the results obtained in test area TS 3 will be examined in detail.

The crest width of TS 3 is 14.8 m (square) and the bottom width 40 m. It is provided with an approximately 10 m wide loading berm, 1.5 m thick. The fill placed on the area amounts to maximum of about 4.2 m, corresponding to a load of about 80 kN/m^2. Owing to submergence of the fill during the course of settlement the load, in kN/m^2, will be reduced successively by about $8s$, where s = settlement, in m, of the soil surface.

The drains, type Mebradrain, were installed to a depth of 12 m in a square pattern with a spacing of 1.0 m which yields $D = 1.13$ m. The equivalent drain diameter determined according to equation (18) becomes $d_w = 0.066$m. The equivalent diameter of the mandrel $d_m = 0.10$ m. The smear zone is estimated at $d_s = 0.20$ m. The permeability ratios k_h/k_s and κ_h/κ_s are assumed equal to the ratio c_h/c_v.

The consolidation characteristics of the clay deposit, determined by oedometer tests, can be summarised as follows (DMJM International, 1996; Airports Authority of Thailand, 1996): average coefficient above the preconsolidation pressure $c_v = 1.06$ m^2/year (standard deviation = 0.061 m^2/year), $c_h = 1.37$ m^2/year (standard deviation = 0.050 m^2/year). This yields $k_h/k_s = \kappa_h/\kappa_s = 1.3$ (in a paper previously published by the author (Hansbo, 1997b) this ratio was assumed equal to 2). The virgin compression ratio CR varies from 0.3 to 0.55 (average 0.43; standard deviation 0.1) and the average recompression ratio $RR = 0.03$ (standard deviation = 0.007). The clay penetrated by the vertical drains is slightly overconsolidated with a preconsolidation pressure about 15–50 kPa higher than the effective overburden pressure. The clay below the tip of the drains is heavily overconsolidated. In this case the influence on the consolidation process of one-dimensional vertical consolidation will be ignored owing to difficulties in assessing the drainage conditions.

The monitoring system consisted of vertical settlement meters placed on the soil surface and at different depths and of inclinometers to study the horizontal displacements. Unfortunately, the results of the settlement observations at various depths are contradictory and, therefore, only the surface settlement observations can be trusted. The contribution to the vertical settlement of horizontal deformations is analysed on the basis of the inclinometers placed 7.8 m from the centre of the test area. Denoting the area created by horizontal deformation versus depth by A, the vertical settlement s owing to the horizontal deformations is calculated as the mean of the two values $4A/14.8$ and $\pi \cdot A/14.8$. The total settlement observed, including the settlement caused by horizontal deformations, and the thus corrected settlement curve, representing merely the effect of consolidation, are shown in Fig.12. A follow-up of the course of consolidation settlement according to Asaoka's method, based on settlement observations at equal time intervals (*cf.* Section 5.2) results in the following relation: $s_i = 0.2625 + 0.8195 s_{i-1}$ which yields the primary settlement $s_p = 1.45$ m.

The settlement analysis based on the λ method is carried out in 4 successive steps: loadstep 1 with load $\Delta q_1 = 20$ kN/m^2; loadstep 2 with load $\Delta q_2 = 30$ kN/m^2; loadstep 3 with $\Delta q_3 = 10$ kN/m^2 and loadstep 4 with $\Delta q_4 = 20$ kN/m^2. The primary consolidation settlement caused by a load intensity of 80 kN/m^2, determined on the basis of the compression characteristics, becomes equal to 1.4 m. By slightly modifying the compression characteristics to yield a final primary consolidation settlement of 1.45 m, we find $\Delta s_1 = 0.15$ m; $\Delta s_2 = 0.6$ m; $\Delta s_3 = 0.2$ m and $\Delta s_4 = 0.5$ m (in total 1.45 m).

The analysis of the consolidation process according to equation (19) has to be carried out in the following way. The degree of consolidation $\overline{U}_1$, inserting $\Delta h_1 = \Delta q_1/\gamma_w$, determines the course of settlement in the first loadstep. When calculating the course of settlement in the second loadstep we have to apply the value $\Delta h_2 = (1-\overline{U}_1)\Delta q_1/\gamma_w + \Delta q_2/\gamma_w$ and the settlement at the end of the loadstep is obtained from $\Delta s = \Delta s_1 \overline{U}_1 + \left[\Delta s_1(1-\overline{U}_1) + \Delta s_2\right]\overline{U}_2$, and so on. Now, 50 days after the start of loading (consolidation time $t = 50 - 15 = 35$ days; $\Delta h = 2$ m; $\lambda = 0.37$ m^2/year) we find $\overline{U} = 0.21$ which yields $s = 0.03$ m. In loadstep 2 the load $\Delta q_2 = 30$ kN/m^2 has to be increased by $0.79 \cdot 20 = 16$ kN/m^2 corresponding to $\Delta h_{2,corr} = 4.6$ m and $\Delta s_{2,corr} = 0.6 + 0.12 = 0.72$ m. After 75 days when loadstep 2 is completed we find $[t = (75-50)/2]$ $\overline{U} = 0.12$ which yields $\Delta s = 0.09$ m and $s = 0.12$ m. After 140 days when loadstep 3 is being applied we have ($t = 12.5 + 65 = 77.5$ days) $\overline{U} = 0.50$ from which $\Delta s = 0.36$ m and $s = 0.39$ m. This yields $\Delta h_{3,corr} = 3.3$ m and $\Delta s_{3,corr} = 0.2 + 0.36 = 0.56$ m. After 220 days when loadstep 4 is being applied we find ($t = 80$ days) $\overline{U} = 0.46$ from which $\Delta s = 0.26$ m and $s = 0.26 + 0.39 = 0.65$ m. This yields $\Delta h_{4,corr} = 1.8+2.0 = 3.8$ m and $\Delta s_{4,corr} = 0.5 + 0.3 = 0.8$ m. After 250 days when loadstep 4 is completed we have ($t = 15$ days) $\overline{U} = 0.13$ which yields $\Delta s = 0.10$ m and $s = 0.75$ m. 100, 200 and 400 days later we have $\overline{U} = 0.59$, $\overline{U} = 0.74$ and $\overline{U} = 0.89$ from which $\Delta s = 0.47$ ($s = 0.47 + 0.65 = 1.12$ m), 0.67 ($s = 1.24$ m) and 0.71 m ($s = 1.36$ m), respectively.

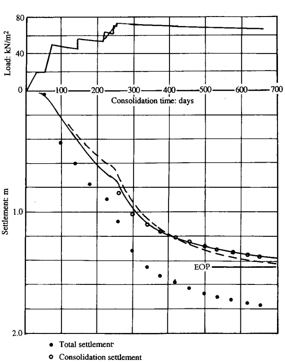

Fig.12. Settlement of ground surface in the Bangkok test field, Thailand. Test area TS 3: 1.0 m drain spacing ($D = 1.13$ m). Settlement corrected with regard to immediate and longterm horizontal displacements. EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (19). Broken lines: analytical results according to equation (9).

The theoretical course of settlement determined in the conventional way is less complicated in that the total consolidation curve can be determined for each loadstep separately and added to each other. Assuming $c_h = 0.93$ m^2/year (in the paper previously mentioned (Hansbo, 1997b) the coefficient c_h was assumed equal to 1.2 m^2/year owing to the fact that the ratio k_h/k_s was put equal to 2 instead of 1.3 now applied) we find, to give an example, 400 days after the start of the loading process ($t_1 = 385$ days, $\overline{U} = 0.92$; $t_2 = 340$ days, $\overline{U} = 0.89$; $t_3 = 260$ days, $\overline{U} = 0.82$; $t_4 = 170$ days, $\overline{U} = 0.67$) the settlement $s = 0.92 \cdot 0.15 + 0.89 \cdot 0.6 + 0.82 \cdot 0.2 + 0.67 \cdot 0.5 = 1.17$ m.

The results obtained by the two methods of analysis are shown in Fig. 12.

Inserting the maximum value $\Delta h = 4.6$ m into equation (26), the values $c_h = 0.93$ m^2/year and $\lambda = 0.37$ m^2/year correspond to $i_l = 3.5$ and $i_{max} = 22.5$.

7.3 The Vagnhärad Vacuum Test

Torstensson (1984) reported an interesting full-scale test in which consolidation of the clay was achieved by the vacuum method. The subsoil at the test site consists of postglacial clay to a depth of 3 m and below this of varved glacial clay to a depth of 9 m underlain by silt. The clay is slightly overconsolidated with a preconsolidation pressure about 5–20 kPa higher than the effective overburden pressure. The coefficient of consolidation c_h was found equal to 0.95 m^2/year and the average virgin compression ratio CR equal to 0.7 (max. 1.0).

The vacuum area, 12 m square, was first covered by a sand/gravel layer 0.2 m in thickness, and then by a Baracuda membrane which was buried to 1.5 m depth along the border of the test area and sealed by means of a mixture of bentonite and silt. Mebradrains ($d_w = 0.066$ m) were installed in a square pattern with 1.0 m spacing to a depth of 10 m. The equivalent diameter of the mandrel $d_m = 0.096$ m. The average underpressure achieved by the vacuum pump was 85 kPa. After 67 days the vacuum process was stopped and then resumed after 6 months of rest. From the shape of the settlement curve (Fig. 13) Asaoka's method yields the correlation $s_i = 0.0756 + 0.9075 s_{i-1}$ from which $s_p = 0.82$ m. This value is low with regard to the loading conditions and the compression characteristics. The main reason seems to be that the applied vacuum effect is not fully achieved in the drains. Thus, the primary settlement 0.82 m corresponds to a vacuum effect of about 35 kPa ($\Delta h = 3.5$ m). Another reason may be that the test area is too small as compared to the thickness of the clay layer.

The theoretical settlement curve in this case has to be determined in two steps, the first one up to a loading time of 67 days leading to a then settlement $s_1 = \overline{U}_{h1} s_p$. In the next loadstep, starting again from the time of resumption of the application of vacuum, the remaining primary settlement is obtained from the relation $\Delta s = \overline{U}_h \left(s_p - s_1\right)$, i.e. the settlement $s_t = s_1 + \overline{U}_{ht} \left(s_p - s_1\right)$ where t starts from the time of resumption of the application of vacuum. In this case, where vacuum is applied to create underpressure in the drains, the effect of vertical one-dimensional consolidation is eliminated.

Inserting the values $D = 1.13$ m, $d_w = 0.066$ m, $d_s = 0.19$ m, $k_h/k_s = \kappa_h/\kappa_s = 4$ and $\Delta h = 3.5$ m into equations (9) and (19), the best agreement between theory and observations is found for $\lambda = 0.95$ m^2/year and $c_h = 2.4$ m^2/year (Fig. 13). Even in this case the λ theory agrees better with observations than the classical theory.

Inserting the maximum value $\Delta h = 3.5$ m into equation (26), the values $c_h = 2.4$ m^2/year and $\lambda = 0.95$ m^2/year correspond to $i_l \approx 14$ and $i_{max} \approx 8$.

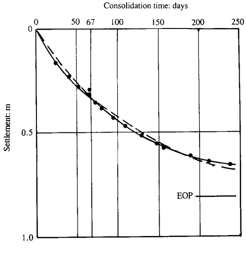

Fig. 13. Results of settlement observations at Vagnhärad, Sweden. Consolidation by vacuum. 1.0 m drain spacing ($D = 1.13$ m). EOP = end of primary consolidation settlement estimated according to Asaoka's method. Full lines: analytical results according to equation (19). Broken lines: analytical results according to equation (9).

7.4 The Arlanda project

The extension of the international airfield at Arlanda, situated some 30 km north of Stockholm, entails, among other things, the construction of a new runway at a site with very bad soil conditions. The soil at the site consists of up to 5 m of peat underlain by high-plasticity, very soft, normally consolidated clay with a maximum thickness of about 10 m. After excavation of the peat layer, preloading has been undertaken both in undrained condition and in combination with vertical drain installations, the latter wherever the thickness of the clay layer exceeds 5 m. The consolidation process is monitored by settlement and pore pressure observations. Mebradrains were installed in equilateral triangular pattern with a drain spacing of 0.9 m. The core of Mebradrains is now equal to the core once used only in Geodrains (see Hansbo, 1981, 1986). Two cases of observations will be presented: one (site K) where the overload consists of 19.5 m sand and gravel ($\Delta q = 390$ kN/m^2) and the other (site L) where the overload consists of 16.2 m sand and gravel ($\Delta q = 325$ kN/m^2).

The soil consists of clay with silt and sand seams, at site K to a depth of 9.7 m (with a sand layer from 1.6 to 1.8 m) and at site L to a depth of 7.8 m. The undrained shear strength of the clay is fairly constant, about 5–10 kPa, irrespective of depth. In order to cope with the influence on the course of settlement of a time-consuming stepwise placement of the load, each break in the rate of loading has been analysed separately on the basis of a direct use of the oedometer curves. At site K the following settlement values Δs were obtained: loadstep 0–80 kN/m^2 $\Delta s_1 = 1.75$ m; loadstep 80–215 kN/m^2 $\Delta s_2 = 0.50$ m; loadstep 215–390 kN/m^2 $\Delta s_3 = 0.30$ m (total primary settlement 2.55 m). At site L the following settlement values Δs were obtained: loadstep 0–80 kN/m^2 $\Delta s_1 = 0.95$ m; loadstep 80–325 kN/m^2 $\Delta s_2 = 0.55$ m (total primary settlement 1.5 m). The settlement values calculated on the basis of the oedometer tests have been checked by Asaoka's method. This results in the correlations $s_i = 0.5265 + 0.7948 s_{i-1}$ at site K and $s_i = 0.3487 + 0.7669 s_{i-1}$ at site L. The primary settlements thus obtained become $s_p = 2.57$ m at site K and $s_p = 1.5$ m at site L.

The coefficient of consolidation c_v according to the oedometer tests varies from about 0.2–0.3 m^2/year at preconsolidation pressure to about 0.5–1.0 m^2/year (maximum 2.5 m^2/year) at the end of primary consolidation under the applied overload. The coefficient of consolidation c_h was not determined.

The loading conditions and the settlement observations in the two cases are shown in Fig. 14. Assuming $d_w = 0.066$ m and $d_s = 0.19$ m, $k_h/k_s = \kappa_h/\kappa_s = 6$ (chosen because of the existence of silt and sand seams in the clay deposit), $l = 4$ m and $c_v = 0.4$ m^2/year the best fit between theory and observations is obtained for $\lambda = 1.6$ m^2/year and $c_h = 4.75$ m^2/year. In this case, as was already demonstrated in Section 7.2, when a great deal of the consolidation process takes place during the loading period, the analysis of the consolidation process according to equation (20) has to be carried out in the following way. The degree of consolidation $\overline{U}_1$, inserting $\Delta h_1 = \Delta q_1/\gamma_w$, determines the course of settlement in the first loadstep. Now, when calculating the course of settlement in the second loadstep we have to apply the value $\Delta h_2 = \left(1 - \overline{U}_1\right)\Delta q_1 / \gamma_w + \Delta q_2 / \gamma_w$ and the settlement at the end of the loadstep is obtained from $\Delta s = \Delta s_1 \overline{U}_1 + \left[\Delta s_1 \left(1 - \overline{U}_1\right) + \Delta s_2\right]\overline{U}_2$, and so on.

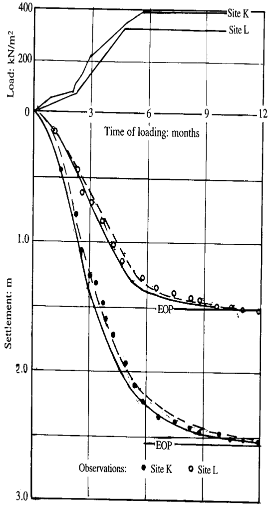

Fig. 14. Settlement of ground surface under fill embankment for a new runway under construction at Arlanda Airport, Stockholm, sites K and L. Drain spacing 0.9 m (D = 0.95 m). Full lines represent analytical results according to equation (20), broken lines analytical results according to equation (11).

The settlement curves are then adjusted for the rate of loading according to the well-known graphical procedure suggested by Terzaghi.

Choosing site K as an example of the analysis that forms the basis of the settlement diagrams shown in Fig. 14, we find, when loadstep 1 is completed, inserting the consolidation time one month (the length of time that corresponds to full loading condition) $\bar{U}_1 = 0.46$ according to equation (20). This yields $s = 0.81$ m and a piezometric head $\Delta h_2 = 0.54 \cdot 8 + 135/10 = 17.8$ m. At the completion of loadstep 2, inserting a time of consolidation of half a month (the time of full loading), we have $\bar{U}_2 = 0.38$ which yields $s = 0.38(2.25 - 0.81) + 0.81 = 1.36$ m. The piezometric head now becomes $\Delta h_3 = 0.62 \cdot 17.8 + 175/10 = 28.5$ m. Finally, at the completion of loadstep 3, when the definite load has been reached, we have (time of consolidation under full load 1 1/4 month) $\bar{U}_3 = 0.70$ which yields $s = 0.70(2.57 - 1.36) + 1.36 = 2.21$ m. Three months later we find $\bar{U}_3 = 0.93$ from which $s = 2.49$ m.

SUMMARY

Results of permeability tests on clay indicating a deviation from Darcy's flow law are unequivocally confirmed by the results of full-scale investigations on consolidation rates obtained in vertical drain projects in different parts of the world. Thus, the consolidation theory developed on the assumption of an exponential correlation between flow velocity v and hydraulic gradient i, below a certain limiting value i_l, originally put forward by the author in 1960, undoubtedly agrees better with case records than the classical consolidation theory based on validity of Darcy's flow law. The value of the exponent n in the exponential flow law $v = \kappa i^n$ can generally be put equal to 1.5 in accordance with the author's original proposal. Only when the maximum hydraulic gradient created by the overload is excessive in relation to the value i_l, limiting the exponential correlation between flow and hydraulic gradient, may the classical consolidation theory give equally good (or possibly even better) agreement with observations.

It should be noticed that the equation based on an exponential correlation between flow rate and hydraulic gradient, governing the consolidation rate in a vertical drain project, is general and can be utilised also when the correlation is linear. The agreement with the classical solution becomes satisfactory if the exponent n is put equal to 1.0001.

In a vertical drain project. the effect on the consolidation process of one-dimensional vertical consolidation in undrained condition is relatively difficult to predict but can generally be neglected. Its contribution can be included by an increase in the coefficient of consolidation to be applied in the vertical drain analysis. However, one has to consider that such a solution can be misleading with respect to the degree of consolidation obtained in the middle of the clay deposit where the effect of vertical one-dimensional consolidation is minimum. From a practical view-point, the design of a vertical drain system has to based on the result achieved in the layer with the lowest coefficient of consolidation and at the depth where the influence of one-dimensional vertical consolidation is at its lowest.

Excess pore pressure observations may seem to be the most logical way of checking the degree of consolidation achieved in a vertical drain project. However, the pore pressures observed may be misleading for several reasons: the position of the piezometer in relation to the drains is uncertain; the pore water pressure may not revert to its original value; etc.

Settlement observations are usually reliable but then the final primary consolidation settlement has to be known. In a test area this does not represent a serious problem since its value can be predicted by the aid of Asaoka's method. However, with regard to the influence of one-dimensional vertical consolidation on the rate of settlement, the designer of a vertical drain system should strive to find the effect on settlement caused by radial drainage only. This is important because of the reasons mentioned above. In practice, this can be done by the use of equations taking into account both radial and vertical pore water flow where the coefficients of consolidation c_h (λ) and c_v are determined by trial and error to give good agreement between theory and observations. Then the effect of radial drainage only is obtained by putting $c_v = 0$.

In cases where occasional overloading is utilised in order to avoid future settlement, the problem when to remove the overload is of paramount interest

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